University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

PROBLEM SET 1

1. Specify the properties of two vectors $\mathbf{a}$ and $\mathbf{b}$ such that

(a.) $\mathbf{a}+\mathbf{b}=\mathbf{c}$ and $|\mathbf{a}|+|\mathbf{b}|=|\mathbf{c}|$.

(b.) $\mathbf{a}+\mathbf{b}=\mathbf{a}-\mathbf{b}$.

(c.) $\mathbf{a}+\mathbf{b}=\mathbf{c}$ and $|\mathbf{a}|^2+|\mathbf{b}|^2=|\mathbf{c}|^2$.

(d.) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$.

(e.) $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|=|\mathbf{b}|$.

2. K&K problem 1.2 "Find the cosine of the angle between $\mathbf{A}=(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ and $\mathbf{B}=(-2\hat{\mathbf{i}}-3\hat{\mathbf{j}}-\hat{\mathbf{k}})$."

3. The relation between Cartesian (x,y,z) and spherical polar (r,θ,ϕ) coordinates is:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta.$$

Consider two points on a sphere of radius R: (R,θ_1,ϕ_1) and (R,θ_2,ϕ_2). Use the dot product to find the cosine of the angle θ_{12} between the two vectors which point to the origin from these two points. You should obtain:

$$\cos\theta_{12} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2).$$

4. New York has North Latitude $(= 90^\circ-\theta)$ $= 41^\circ$ and West Longitude $(= 360^\circ-\phi) = 74^\circ$. Sydney has South Latitude $(= \theta-90^\circ) = 34^\circ$ and East Longitude $(= \phi) = 151^\circ$. Take the earth to be a sphere of radius 6370 km; use the result of Problem 3.

(a.) Find the length in km of an imaginary straight tunnel bored between New York and Sydney.

(b.) Find the distance of the shortest possible low-altitude flight between the two cities. (*Hint:* The "great circle" distance along the surface of a sphere is just $R\theta_{12}$, where θ_{12} is the angle between the two points, measured in radians.)

5. K&K problem 1.6 "Prove the law of sines using the cross product. It should only take a couple of lines. (*Hint*: Consider the area of a triangle formed by $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, where $\mathbf{A}+\mathbf{B}+\mathbf{C}=0$.)"

6. K&K problem 1.11 "Let $\mathbf{A}$ be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\mathbf{A} = (\mathbf{A}\cdot\hat{\mathbf{n}})\,\hat{\mathbf{n}}+(\hat{\mathbf{n}}\times\mathbf{A})\times\hat{\mathbf{n}}$."

7. If the air velocity (velocity with respect to the air) of an airplane is $\mathbf{u}$, and the wind velocity with respect to the ground is $\mathbf{w}$, then the ground velocity $\mathbf{v}$ of the airplane is

$$\mathbf{v} = \mathbf{u}+\mathbf{w}.$$

An airplane files a straight course (with respect to the ground) from P to Q and then back to P, with air speed $|\mathbf{u}|$ which is always equal to a constant U_0, regardless of the wind. Find the time required for one round trip, under the following conditions:

(a.) No wind.

(b.) Wind of speed W_0 blowing from P to Q.

(c.) Wind of speed W_0 blowing perpendicular to a line connecting P and Q.

(d.) Wind of speed W_0 blowing at an angle θ from a line connecting P and Q.

(e.) Show that the round trip flying time is always least for part (a.).

(f.) What happens to the answers to (b.)-(d.) when $W_0 > U_0$? Interpret this limiting condition physically.

8. A particle moves along the curve $y = Ax^2$ such that its x position is given by $x = Bt$ (t = time).

(a.) Express the vector position $\mathbf{r}(t)$ of the particle in the form

$$\mathbf{r}(t) = \mathbf{i}f(t)+\mathbf{j}g(t) \;\; [\text{or } \hat{\mathbf{x}}f(t)+\hat{\mathbf{y}}g(t)]$$

where $\mathbf{i}$ and $\mathbf{j}$ [or $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$] are unit vectors, and $f(t)$ and $g(t)$ are functions of t.

(b.) Find the (vector) velocity $\mathbf{v}(t)$ as a function of t.

(c.) Find the (vector) acceleration $\mathbf{a}(t)$ as a function of t.

(d.) Find the (scalar) speed $|\mathbf{v}(t)|$ as a function of t.

(e.) Find the (vector) average velocity $\langle \mathbf{v}(t_0) \rangle$ between $t = 0$ and $t = t_0$ where t_0 is any positive time.

9. Below are some measurements taken on a stroboscopic photograph of a particle undergoing accelerated motion. The distance s is measured from a fixed point, but the zero of time is set to coincide with the first strobe flash:

time (sec)	distance (m)
0	0.56
1	0.84
2	1.17
3	1.57
4	2.00
5	2.53
6	3.08
7	3.71
8	4.39

Plot a *straight-line* graph, based on these data, to show that they are fitted by the equation

$$s = a(t - t_0)^2/2,$$

where a and t_0 are constants, and extrapolate the line to evaluate t_0.